arXiv:0705.0845v2 [gr-qc] 25 Jan 2008

The Quantum World is an AdS$_5$ with the Quantum Relativity Symmetry $SO(2,4)$

Otto C. W. Kong

Department of Physics and Center for Mathematics and Theoretical Physics,

National Central University,

Chung-li, Taiwan 32054

Abstract

Quantum relativity as a generalized, or rather deformed, version of Einstein relativity with a linear realization on a classical six-geometry beyond the familiar setting of space-time offer a new framework to think about the quantum space-time structure. The formulation requires two deformations to be implemented through imposing two fundamental invariants. We take them to be the independent Planck mass and Planck length. Together, they gives the quantum $\hbar$. The scheme leads to $\boldsymbol{SO(2,4)}$ as the relativity symmetry. The quantum world has an AdS$_5$ 'classical' geometry, which is parallel to the "conformal universe", but not scale invariant.

PACS numbers:

Introduction :-   This letter is in a way a sequel of Ref.[1], in which an interesting new perspective to think about the subject of Quantum Relativity has been introduced. Since the early days of the quantum theory, the notion about some plausible 'quantum structure' of space-time itself at the microscopic limit, in connection to quantum gravity or otherwise, has been pondered by many theorists. The analysis of Ref.[1] leads to a radical proposal for the description of space-time at the microscopic or quantum level — that it has to be described as part of something bigger, here we dub the 'quantum world', with dimensions beyond space and time. Authors of Ref.[1] wanted to present a detailed and clear account of theoretical formulation and thinking behind the radical proposal. We hope this letter can bring that perspective to the attention of a much broader audience.

On the other hand, we do have a very important conceptual breakthrough, at least within the subject domain of Quantum Relativity, to be presented here. It leads to an important new result — the identification of the quantum world as an AdS_5 with $\boldsymbol{SO(2,4)}$ as the relativity symmetry. Readers familiar with some of the more popular themes in the literature going under the title of AdS-CFT correspondence [2] and the holographic principle [3] may incline to consider the particular result as what has been well known. The $\boldsymbol{SO(2,4)}$ group algebra has been discussed as the symmetry of 'quantum gravity' in six dimensions. The relevance of $\boldsymbol{SO(2,4)}$ and AdS_5 to quantum gravity obtained from the two very different approaches may suggest both are likely to be getting something right about Nature. However, the connection really ends there. The two approaches are logically completely independent. The study of AdS_5 gravity is basically taking AdS_5 as the space-time, or rather part of an $AdS_5 \times S_5$ compactification of the ten dimensional superstring theory. One can justify the introduction of AdS_5 somewhat independent of the 'string' assumption, based on the closely related perspective of Yang-Mills theory at the large-N limit. Our approach here, however, has nothing to do with any of that. While many theorists may believe in string theory or large-N Yang-Mills theory to be useful in describing Nature, one should bear in mind that we have no experimental evidence of that. String theory still has to produce an experimental verifiable prediction; and the Yang-Mills theories that have been established experimentally have only $N \leq 3$. Our approach, arguably, starts with a much more minimal set of theoretical assumptions. We look for a direct description of quantum physics (of the classical notion of space-time), as for example also adopted in Ref.[9]. This is in contrast to the 'quantization perspective', adopted for instance in most

studies within the string theory framework, within which one finds a 'classical' fundamental description and produce the quantum counterpart through some quantization procedure. Ref.[9] starts with a new, supposed to be more fundamental dynamic principle, assuming a (matrix model) noncommutative geometry. We start with a new relativity symmetry and obtain a dual classical and noncommutative geometric description, before any consideration of dynamics. One should also note that works along our perspective is only at a very preliminary stage. We believe it deserves a good chance to be seriously considered and developed.

Physicists know of two relativities, namely Galilean relativity of Newtonian physics and Einstein relativity. Interesting enough, going from the former to the latter can be considered a direct result of the mathematical procedure of symmetry deformation or stabilization [4]. Such stabilizations may be considered as the only legitimate symmetries to describe physics as confirming an unstable symmetry to be the 'correct' symmetry requires infinite experimental precision, establishing $1/c^2$ as exactly zero in this case. The same procedure applied again (to the Poincaré symmetry) with minimal physics inputs leads a new relativity. Taking notion from quantum physics, such as the existence of a fundamental Planck scale, leads to a new relativity to be considered as the Quantum Relativity. The approach is actually technically quite simple and considered to be accessible to a broad range of physicists beyond that of the 'high energy theorists'.

Quantum relativity :- To get a better idea of what symmetry deformation is all about, let us take a look at the Galilean to Einstein case. Think about the algebra of the generators for Galilean boosts. Deforming the zero commutators of any two Galilean boosts to $1/c^2$ times a corresponding rotation generator gives the Lorentzian $\boldsymbol{SO(1,3)}$ symmetry. If there has to be a velocity with magnitude c invariant under reference frame transformations, the above is the unavoidable mathematical consequence. The right physics interpretation says that we should think about 4D (Minkowski) space-time instead of 3D space as the basic arena for fundamental physics with Lorentzian, or Poincaré, symmetry.

The idea of a quantum relativity dates back more than half a century [5]. A simple way to put it is to say that if quantum physics introduces the idea of the Planck scale, one may want it to be characterized by a reference frame independent quantity. For instance, you do not want to see the Planck length to suffer from a Lorentz contraction. It has been realized that that can only be done by modifying, or rather deforming, the relativity symmetry, basically

in the same way as deforming the Galilean $ISO(3)$ algebra to the Lorentzian $SO(1,3)$. The first symmetry for such a quantum relativity suggested was essentially $SO(1,4)$ [6], though some recent authors bringing back this old topic preferred to think about it outside the Lie algebra framework [7, 8]. Sticking to the Lie algebra deformation perspective [4], Ref.[1] gives a very radical but otherwise sensible physics picture of the quantum relativity through a linear realization. There, the quantum relativity symmetry was identified as $SO(1,5)$, through one further deformations as suggested in so-called triply deformed special relativity [8]. The three deformations, from the Galilean relativity, are parametrized by the speed of light c, the Planck scale as a mass-energy scale κc as originated in Ref.[5], and a sort of infrared ("length") bound associated with the cosmological constant [8]. The radical physics picture follows naturally the lesson from Einstein. Just like the 3D space is part of the 4D space-time, the 4D space-time is then part of the quantum world — a (classical) 6D geometry with the two extra dimensions being something beyond space and time!

Here in this letter, we present rather $SO(2,4)$ as the symmetry for the quantum relativity. It is still a three deformation setting, but the last deformation is done differently. The latter is still implemented through a limiting length, but rather on the ultra-violet. It is the Planck length, ℓ. The starting point is the important observation that implementing a Planck mass is not enough to get the relativity quantum, because there is no $\hbar$. In fact, interpreting Planck energy and Planck length as essentially one (the Planck) scale assumes quantum physics. Formulating a quantum relativity should rather be trying to get that as a result. We have to produce explicitly the $\hbar$ as an invariant ! The current letter present exactly such a formulation, getting $\hbar$ as $\kappa c \ell$. This is breaking with the unquestioned tradition since 1947! An AdS_5 hypersurface within the six-geometry is obtained as the quantum world. We will also discuss some physics features of the new $SO(2,4)$ relativity.

We have discussed briefly the deformation of the Galilean $ISO(3)$ the $SO(1,3)$ Lorentz symmetry. The mathematics of further deformations are basically the same. We skip most of the explicit details, but summarize in Table 1 the essential aspects of the three deformations. Note that $\eta_{\mathcal{MN}} = (1,-1,-1,-1,-1,1)$ with the indices go from $\mathbf{0}$ to $\mathbf{5}$; η_{AB} is the $\mathbf{0}$ to $\mathbf{4}$ part; other than that, it is the usual notation. $J_{\mathcal{MN}}$ here denotes the 15 generators of the the $SO(2,4)$ algebra accordingly.

With 4D translations included, the Poincaré symmetry $ISO(1,3)$ resulted is again unstable

against deformation. The stabilization is either $SO(1,4)$ or $SO(2,3)$. The Snyder suggestion of the Planck mass as a limiting energy-momentum leads to $SO(1,4)$, to be linearly realized on a 5D geometry[1]. As illustrated in the table, $E^2 - |\vec{p}|^2 c^2 \leq \kappa^2 c^4$ suggests the momentum five-vector $\vec{\pi}^5$ invariant under $SO(1,4)$. Taking the bound as $|\vec{p}|^2 c^2 - E^2 \leq \kappa^2 c^4$ can be easily seen to give $SO(2,3)$ instead. The latter case is obviously of no interest. The new extra coordinate of the 5D geometry is to be identified as an parameter σ external to space-time giving, within $ISO(1,3)$ picture, translations by $p^\mu \sigma$ (c.f. $\Delta x = v^i t$). The σ-coordinate has a spatial geometric signature and a Einstein limit of proper time divided by rest mass[1]. It is neither space nor time.

It looks like the 5D geometry should also admits translational symmetries. The $ISO(1,4)$ resulted is again mathematically unstable. On the other hand, having Planck mass as an invariant may not be enough to get the relativity to describe a quantum world. In fact, without presuming the quantum $\hbar$, the Planck length ℓ is an independent quantity. In the table, we also show the deformation of $ISO(1,4)$ to $SO(2,4)$ based on further imposing ℓ as an 'length' bound on the ultra-violet. The choice of $|z^A| \leq i\,\ell$ with z^A as a 'length' or 'location' vector of the 5D geometry gives a six-vector description of the 'generalized space-time location' $\vec{X}^6$ as an element of the AdS$_5$ geometry. Note that the i, explicitly, in $|z^A| \leq i\,\ell$ means

$$(z^0)^2 - |\vec{z}|^2 - (z^4)^2 \leq -\ell^2 \ ,$$

hence an effective lower bound on *length* — naively $|\vec{z}| \geq \ell$ for $z^0 = z^4$ (say both zero). That is what is in line with the idea of a Planck length. To look at the whole thing from this perspective,

TABLE I: The Three Deformations Summarized:-

$\Delta x^i(t) = v^i \cdot t$	$\Delta x^\mu(\sigma) = p^\mu \cdot \sigma$	$\Delta x^A(\rho) = z^A \cdot \rho$
$|v^i| \leq c$	$|p^\mu| \leq \kappa\,c$	$|z^A| \leq i\,\ell$
$-\eta_{ij} v^i v^j = c^2 \left(1 - \frac{1}{\gamma^2}\right)$	$\eta_{\mu\nu} p^\mu p^\nu = \kappa^2 c^2 \left(1 - \frac{1}{\Gamma^2}\right)$	$\eta_{AB} z^A z^B = -\ell^2 \left(1 + \frac{1}{G^2}\right)$
$M_{0i} \equiv N_i \sim P_i$	$J_{\mu 4} \equiv O_\mu \sim P_\mu$	$J_{A5} \equiv O'_A \sim P_A$
$[N_i, N_j] \longrightarrow -i\,M_{ij}$	$[O_\mu, O_\nu] \longrightarrow i\,M_{\mu\nu}$	$[O'_A, O'_B] \longrightarrow i\,J_{AB}$
$\vec{u}^4 = \frac{\gamma}{c}(c, v^i)$	$\vec{\pi}^5 = \frac{\Gamma}{\kappa\,c}(p^\mu, \kappa\,c)$	$\vec{X}^6 = \frac{G}{\ell}(z^A, \ell)$
$\eta_{\mu\nu} u^\mu u^\nu = 1$	$\eta_{AB} \pi^A \pi^B = -1$	$\eta_{\mathcal{MN}} X^{\mathcal{M}} X^{\mathcal{N}} = -1$
$\mathbb{R}^3 \to SO(1,3)/SO(3)$	$\mathbb{R}^4 \to SO(1,4)/SO(1,3)$	$\mathbb{R}^5 \to SO(2,4)/SO(1,4)$

the complex i for quantum physics comes from the fact that the Planck scales as two, rather than one, invariant quantities (like and beyond c) have to be imposed as bounds on the part of the corresponding vectors with a space-like and a time-like signature, respectively. It is actually the "Minkowski" structure of the classical (six-) geometry that is the true origin of the quantum i.

A strong advantage of having the last deformation achieved through imposing a 'length' bound is the fact that simple translations on the resulted six-geometry cannot be admissible symmetries. That terminates further extension to the unstable $ISO(2,4)$ which will require further deformation according to the philosophy behind the approach. Note that our extra dimensional coordinates are not part of the space-time description at all at the classical limit. Dynamics as behaviors concerning changes of spatial locations or configurations with respect to time has apparently no role for such coordinates. How to think about dynamics at the $SO(2,4)$ invariant setting is a complete open question at this point.

The geometry and the scale/conformal transformation:- Similar to the case for dS$_5$ discussed in Ref.[1], the set of z^A's simply give a (Beltrami-type) five-coordinate description of the AdS$_5$ hypersurface $\eta_{\mathcal{MN}} X^{\mathcal{M}} X^{\mathcal{N}} = -1$. In terms of z^A, the metric is given by $g_{AB} = \frac{G^2}{\ell^2}\eta_{AB} + \frac{G^4}{\ell^4}\eta_{AC}\eta_{BD}z^C z^D$. Introducing $q_A \equiv i\hbar \frac{\partial}{\partial z^A}$, and the Lorentzian 5-vectors $Z_A^{(\mathcal{L})} \equiv \eta_{AB} z^B = -\frac{\ell^2}{G^4}z_A$ and $P_A^{(\mathcal{L})} = q_A + Z_A^{(\mathcal{L})} \frac{1}{\ell^2}\left(\eta^{BC} Z_B^{(\mathcal{L})} q_C\right)$, we have representations of the $SO(2,4)$ generators given as

$$J_{\mathcal{MN}} = X_{\mathcal{M}} P_{\mathcal{N}} - X_{\mathcal{N}} P_{\mathcal{M}} = Z_{\mathcal{M}}^{(\mathcal{L})} q_{\mathcal{N}} - Z_{\mathcal{N}}^{(\mathcal{L})} q_{\mathcal{M}} = Z_{\mathcal{M}}^{(\mathcal{L})} P_{\mathcal{N}}^{(\mathcal{L})} - Z_{\mathcal{N}}^{(\mathcal{L})} P_{\mathcal{M}}^{(\mathcal{L})} ; \qquad (1)$$

$P_{\mathcal{M}} \equiv i\hbar \frac{\partial}{\partial X^{\mathcal{M}}}$, and we adopt the natural extended definitions $Z_5^{(\mathcal{L})} \equiv \ell$ and $P_5^{(\mathcal{L})} \equiv q_5 + Z_5^{(\mathcal{L})} \frac{1}{\ell^2}\left(\eta^{BC} Z_B^{(\mathcal{L})} q_C\right) = 0$.

The 'Lorentzian' 5-momentum $P_A^{(\mathcal{L})} = -\frac{1}{\ell} J_{A5}$ is a quantum, noncommutative, generalization of the 'classical' 5-momentum at the level of the intermediate $SO(1,4)$ relativity [1], essentially as introduced by Gürsey [12]. Moreover, its first four components transform as that of a 4-vector under the 4D Lorentz group $SO(1,3)$. We also have

$$[Z_A^{(\mathcal{L})}, P_B^{(\mathcal{L})}] = -i\hbar\, \eta_{AB} . \qquad (2)$$

Another note worthy feature here is that $q_5 = -\frac{1}{\ell}\left(\eta^{BC} Z_B^{(\mathcal{L})} q_C\right)$ resembles the conformal symmetry (scale transformation) generator for the five-geometry with an otherwise Minkowski metric. Translation along z^5 $(= \ell)$ is indeed a scaling of $X^{\mathcal{M}}$. We explore another connection to 4D conformal symmetry below.

In the quantum regime, what one observes depends on the energy scale the system is being probed. For high energy theorists, the importance of the renormalization group evolutions cannot be over-estimated. A quantum frame of reference will likely have to be characterized also by the energy scale as the renormalization scale, or some generalization of that. What is remarkable is that the $SO(2,4)$ symmetry for the relativity is mathematically the same group for conformal symmetry in 4D space-time, usually considered as the symmetry for a scale invariant theory. Our question here is how the relativity symmetry $SO(2,4)$ can be connected to the 4D conformal symmetry $SO(2,4)$, and what that may teach us about the physics of the Quantum Relativity.

Following Ref.[1] (see also Refs.[6, 8]) and discussion above, we write our quantum relativity algebra as :

$$[M_{\mu\nu}, M_{\lambda\rho}] = i\hbar \left(\eta_{\nu\lambda} M_{\mu\rho} - \eta_{\mu\lambda} M_{\nu\rho} + \eta_{\mu\rho} M_{\nu\lambda} - \eta_{\nu\rho} M_{\mu\lambda} \right) ,$$

$$[M_{\mu\nu}, \hat{P}_\lambda] = i\hbar \left(\eta_{\nu\lambda} \hat{P}_\mu - \eta_{\mu\lambda} \hat{P}_\nu \right) ,$$

$$[M_{\mu\nu}, \hat{X}_\lambda] = i\hbar \left(\eta_{\nu\lambda} \hat{X}_\mu - \eta_{\mu\lambda} \hat{X}_\nu \right) ,$$

$$[\hat{X}_\mu, \hat{X}_\nu] = \frac{i\hbar}{\kappa^2 c^2} M_{\mu\nu} , \qquad [\hat{P}_\mu, \hat{P}_\nu] = -\frac{i\hbar}{\ell^2} M_{\mu\nu} ,$$

$$[\hat{X}_\mu, \hat{P}_\nu] = -i\hbar\, \eta_{\mu\nu} \hat{F} , \qquad [\hat{X}_\mu, \hat{F}] = \frac{-i\hbar}{\kappa^2 c^2} \hat{P}_\mu , \qquad [\hat{P}_\mu, \hat{F}] = \frac{-i\hbar}{\ell^2} \hat{X}_\mu , \qquad (3)$$

$(\hbar = \kappa c \ell)$. This is to be matched to the standard form

$$[J_{\mathcal{R}\mathcal{S}}, J_{\mathcal{M}\mathcal{N}}] = i\hbar \left(\eta_{\mathcal{S}\mathcal{M}} J_{\mathcal{R}\mathcal{N}} - \eta_{\mathcal{R}\mathcal{M}} J_{\mathcal{S}\mathcal{N}} + \eta_{\mathcal{R}\mathcal{N}} J_{\mathcal{S}\mathcal{M}} - \eta_{\mathcal{S}\mathcal{N}} J_{\mathcal{R}\mathcal{M}} \right) , \qquad (4)$$

$J_{\mathcal{M}\mathcal{N}} = i\hbar \left(x_{\mathcal{M}} \partial_{\mathcal{N}} - x_{\mathcal{N}} \partial_{\mathcal{M}} \right)$. We identify

$$J_{\mu 4} \equiv -\kappa\, c\, \hat{X}_\mu = i\hbar \left(x_\mu \partial_4 - x_4 \partial_\mu \right) ,$$

$$J_{\mu 5} \equiv -\ell\, \hat{P}_\mu = i\hbar \left(x_\mu \partial_5 - x_5 \partial_\mu \right) ,$$

$$J_{45} \equiv \kappa c \ell \hat{F} = i\hbar \left(x_4 \partial_5 - x_5 \partial_4 \right), \quad J_{\mu\nu} \equiv M_{\mu\nu}. \qquad (5)$$

The result gives an interesting interpretation as suggested by the notation that the generators represent a form of 4D noncommutative geometry. The sets of $\hat{X}_\mu$'s and $\hat{P}_\mu$'s give natural quantum generalizations of the classical x_μ's and p_μ's (represented as $i\hbar\partial_\mu$'s here), or $Z_\mu^{(\mathcal{L})}$'s and q_μ's in term of the five-geometry as discussed above. One can check that they do have the right classical limit. Note that the algebra may also be interpreted as coming from the stabilization of the 'Poincaré + Heisenberg' algebra with $\hat{F}$ being the central generator being deformation. On the AdS$_5$, $-\kappa\, c\, \hat{F}$ is $P_4^{(\mathcal{L})}$, the fifth 'momentum' component.

7

We introduce of the coordinates $x_+ = (x_5 + x_4)/\sqrt{2}$ and $x_- = (x_5 - x_4)/\sqrt{2}$, to be called conformal cone coordinates. The generators $J_{\mu 4}$ and $J_{\mu 5}$ may be replaced by the equivalent set

$$J_{\mu\pm} \equiv i\hbar\,(x_\mu \partial_\pm - x_\pm \partial_\mu) = (J_{\mu 5} \pm J_{\mu 4})/\sqrt{2}\,, \tag{6}$$

where $\partial_\pm = (\partial_5 \pm \partial_4)/\sqrt{2}$, and $J_{+-} \equiv i\hbar\,(x_+\partial_- - x_-\partial_+) = J_{45}$. Mathematical structure of the algebra for conformal symmetry in 4D Minkowski space-time [2] can be obtained through the identification

$$K_\mu \Rightarrow \sqrt{2}J_{\mu-}\,, \qquad P_\mu \Rightarrow \sqrt{2}J_{\mu+}\,, \qquad D \Rightarrow -J_{45}\,. \tag{7}$$

However, the physics picture is to be given by the definitions

$$P_\mu = i\hbar\,\partial'_\mu \equiv i\hbar\,\frac{\partial}{\partial y^\mu}\,, \qquad\qquad D = i\hbar\,y^\mu \partial'_\mu\,,$$
$$K_\mu = i\hbar\,(2y_\mu\,y^\nu \partial'_\nu - y^2\,\partial'_\mu)\,, \tag{8}$$

where y^μ represents the 4-coordinate of Minkowski space-time. Recall that the introduction of the invariant length ℓ admits a description of the coordinate variable x as a pure number (denoted rather by X above). Obviously, the standard 6-coordinate definition for $J_{\mathcal{MN}}$ is invariant under such re-scaling. Next, we consider the 6- to 4- coordinate transformation on a special 4D hyper-surface to be given by the dimensionless $(x^\mu, x^4, x^5) = (y^\mu, \frac{1}{2}\eta_{\mu\nu}y^\mu y^\nu + \frac{1}{2}, \frac{1}{2}\eta_{\mu\nu}y^\mu y^\nu - \frac{1}{2})$. One easily sees that the metric in terms of y^μ is still $\eta_{\mu\nu}$, hence Minkowski. Moreover, we have

$$x_+ = x^- = -1/\sqrt{2}\,, \qquad x_- = x^+ = y^2/\sqrt{2}\,,$$
$$\partial_+ = 0\,, \qquad \partial_5 = -\partial_4 = \frac{1}{\sqrt{2}}\,\partial_- = x^\nu \partial_\nu\,. \tag{9}$$

The latter does give exactly Eq.(8) through expression (7). So, we can say that for the 4D hypersurface in the six-geometry satisfying Eq.(9), translations along x_4 and x_5 do correspond to scaling, as $i\hbar\,\partial_5 = -i\hbar\,\partial_4 = D$. We call this hypersurface the conformal universe. The latter satisfies $\eta_{\mathcal{MN}}x^\mathcal{M}x^\mathcal{N} = 0$ while the quantum world has $\eta_{\mathcal{MN}}x^\mathcal{M}x^\mathcal{N} = -1$ as shown in Table 1. The $J_{\mathcal{MN}}$ transformations of $SO(2,4)$ leaves both invariant. We are then forced to conclude that the quantum world cannot have 4D scale invariance. The analysis also illustrates that translations along x_4 and x_5 can be considered as some sort of scaling, or transformation (energy) scale. To establish the latter on a more solid setting, we do need to first build a theory of dynamics, which is beyond the scope of the present letter.

Conclusion :- Special Einstein relativity as given by $SO(1,3)$ is the deformation or stabilization of the Galilean $ISO(3)$. Along the same line, extending to $ISO(1,3)$ and stabilizing to $SO(1,4)$

has been considered as admitting the deforming parameter $\frac{1}{\kappa^2 c^2}$ to be nonzero. While this gives the finite Planck mass $\boldsymbol{\kappa c}$, there is still no $\hbar$. Going further to $\boldsymbol{ISO(1,4)}$ and then $\boldsymbol{SO(2,4)}$ may be taken as admitting independently the finite Planck length $\boldsymbol{\ell}$. The latter together with $\boldsymbol{\kappa c}$ gives $\hbar$. The symmetry for quantum relativity is hence $\boldsymbol{SO(2,4)}$, the linear realization of which tells that the quantum world is an AdS$_\mathbf{5}$ sitting inside a classical six-geometry of four space-time plus two extra coordinates. $\boldsymbol{ISO(2,4)}$ is not a symmetry for the AdS$_\mathbf{5}$, hence no further extension and deformation. The formulation also gives a quantum, noncommutative, 4D space-time description, fitting well with the natural perspective from the deformation approach that these extra coordinates are neither space nor time. They are connected to the concept of (energy) scale, though the quantum world is not scale invariant but rather 'parallel' to the conformal universe.

The relativity symmetry stabilization approach, with the quite minimal physics input of having the fundamental constants Planck mass and Planck length (hence also the quantum $\hbar$) being the deformation parameters is illustrated to give an AdS$_\mathbf{5}$ as the quantum world with $\boldsymbol{SO(2,4)}$ as the reference frame transformation symmetry. That is but all kinematics, the next challenge is to build a theory of dynamics, or a theory that does give us dynamics as we know it at the classical space-time limit. The latter represents further big challenges to our conceptual thinking about fundamental physics.

---

[1] A. Das and O.C.W. Kong, Phys. Rev. D**73**, 124029 (2006).

[2] O. Aharony *et.al*, Phys. Rep. **323**, 183 (2000).

[3] R. Bousso, Rev. Mod. Phys. **74**, 825 (2002).

[4] R.V. Mendes, J. Phys. A, **27**, 8091 (1994); C. Chryssomalakos and E. Okon, Int. J. Mod. Phys.D **13**, 2003 (2004).

[5] H.S. Snyder, Phys. Rev. **71**, 38 (1947); see also C.N. Yang, Phys. Rev. **72**, 874 (1947).

[6] C. Chryssomalakos and E. Okon, Int. J. Mod. Phys.D**13**, 1817 (2004).

[7] G. Amelino-Camelia, Phys. Lett.B **510**, 255 (2001); Int. J. Mod. Phys.D **11**, 35 (2002); J. Magueijo and L. Smolin, Phys. Rev. Lett. **88**, 190403 (2002); Phys. Rev. D**67**, 044017

(2003).

[8] J. Kowalski-Glikman and L. Smolin, Phys. Rev.D **70**, 065020 (2004).

[9] S.L. Adler, *Quantum Theory as an Emergent Phenomenon*, Cambridge University Press 2004.

[10] J. Kowalski-Glikman and S. Nowak, Class. Quantum Grav. **20**, 4799 (2003).

[11] J. Kowalski-Glikman and S. Nowak, Int. J. Mod. Phys.D **12**, 299 (2003).

[12] F. Gürsey, *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gordon and Breach, New York, 1964), p. 365.